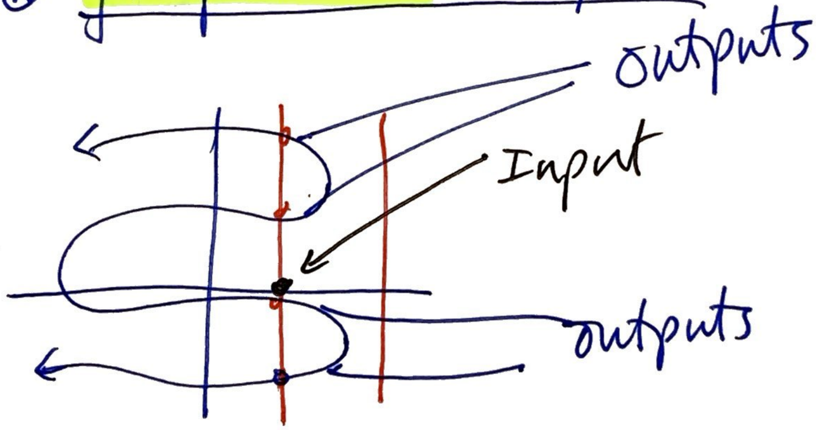
1.1 Functions

- Relationship "maps" the input an object {number or function or ...} and produces another object {number, function, etc}

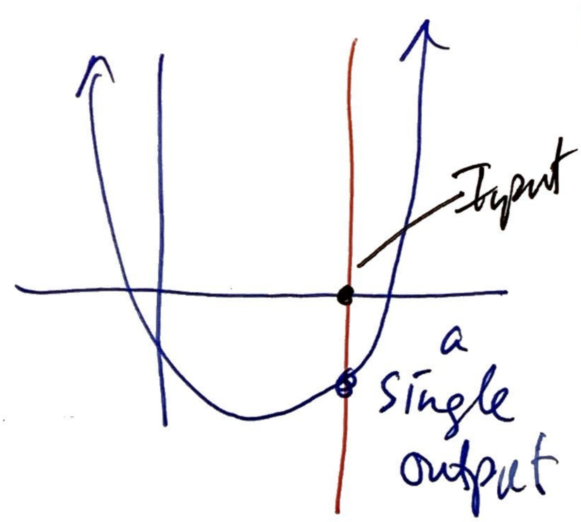
- A function is a relation that provides only a single output for each input

- A one-to-one function is a function that yeilds one input for each chosen output.

graphical example

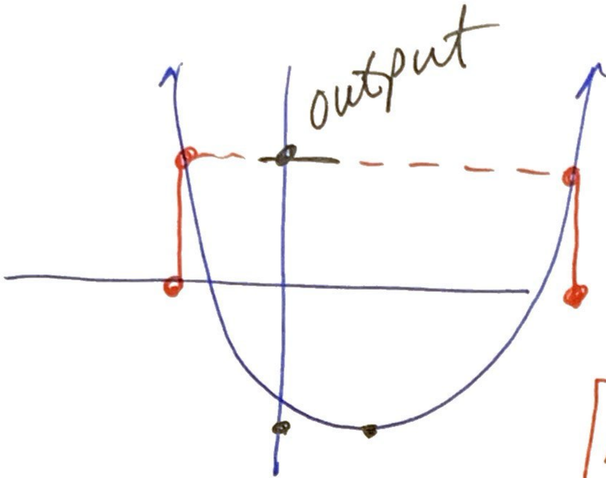


Fails vertical line test ... Not a function



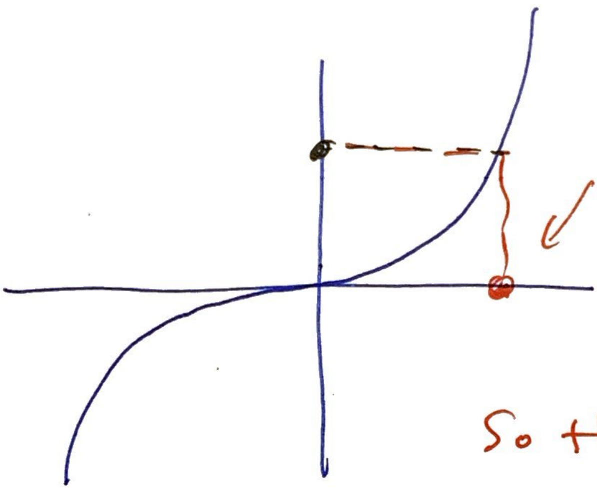
So this is the graph of a function

• A 1-1 function has only one possible choice of input given an output.



Two inputs yeild the same output

$\boxed{\text{Not } 1-1}$



the only input that results in a choosen output

So this $\boxed{\text{Is } 1 \text{ to } 1}$

Table **Ex**

x	$R(x)$
-5	11
-4	7
-3	-6
-2	6
0	0
1	-7
2	11

A function because each input, x, has an output $R(x)$

So the table represents a function

Q: 1-1? The output 11 has two inputs that result in 11

So: $\boxed{\text{NOT } 1-1}$

A) Analytical relations

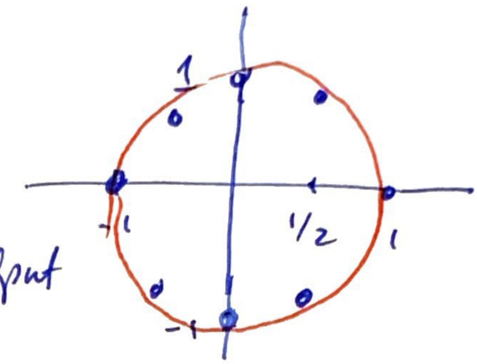
analytical

$x^2 + y^2 = 1$

x	$y = \pm\sqrt{1-x^2}$
-10	$y = \pm\sqrt{1-10^2} = \pm\sqrt{-99}$
-1	$y = \pm\sqrt{1-(-1)^2} = 0$
$1/2$	$y = \pm\sqrt{1-(\frac{1}{2})^2} = \pm\sqrt{3/4} = \pm\frac{\sqrt{3}}{2} \approx \pm 0.7$
0	$y = \pm 1$
$-\frac{1}{2}$	$y = \pm\frac{\sqrt{3}}{2}$
1	$y = 0$

No output

graph



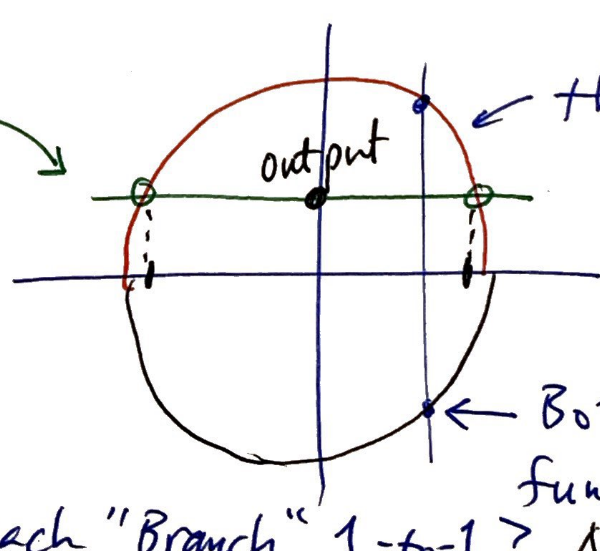
two outputs for each input

$\Rightarrow$ $\boxed{\text{Not a function}}$

- Treat each part (+/-) separately

$\boxed{y = +\sqrt{1-x^2}}$ or $\boxed{-\sqrt{1-x^2}}$

Horizontal Line Test for graphs



this is a function (top only)

Bottom is a function also

Q: Is each "Branch" 1-to-1? Not

⊛ Analytical function

- we use letters to discuss an arrangement of input variables, a.k.a. <u>Independent Variable</u> typically "x"

$$f(x) = 3x^2 - 2x + 1$$

The generic form can leave the symbol out

$$\boxed{f(\ ) \equiv 3(\ )^2 - 2(\ ) + 1}$$

ex

- Feed "f" an x:

$$f(x) = 3(x)^2 - 2(x) + 1$$

- Feed "f" a number like 11

$$f(11) = 3(11)^2 - 2 \cdot 11 + 1$$
$$= \underline{\underline{342}}$$

- Feed "f" x^3

$$f(x^3) = 3(x^3)^2 - 2(x^3) + 1$$
$$= \boxed{3x^6 - 2x^3 + 1}$$

$\boxed{\text{EX}}$ Form $\dfrac{f(x+h)-f(x)}{h}$

$$= \frac{\left[3(x+h)^2 - 2(x+h) + 1\right] - \left[3x^2 - 2x + 1\right]}{h}$$

$$= \frac{\left(3(x^2+2hx+h^2) - 2x - 2h + 1\right) - \left(3x^2 - 2x + 1\right)}{h}$$

$$= \frac{\cancel{3x^2} + 6hx + 3h^2 - \cancel{2x} - 2h + \cancel{1} - \cancel{3x^2} + \cancel{2x} - \cancel{1}}{h}$$

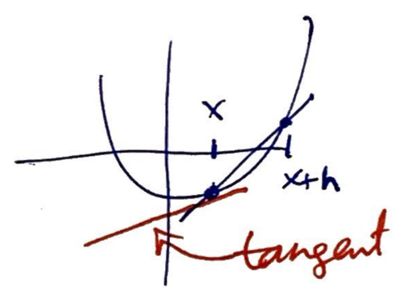
$$= \frac{6hx + 3h^2 - 2h}{h}$$

$$= \underline{\underline{6x - 2 + 3h}}$$

BTW: if we take h to be smaller and smaller

then we say

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$



$= \underline{\underline{6x-2}}$ this is the derivative of

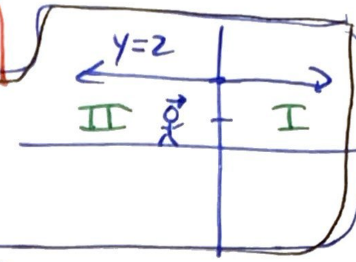
$f(x) = 3x^2 - 2x + 1$

Basic Functions

Constant functions

$f(x) = c$

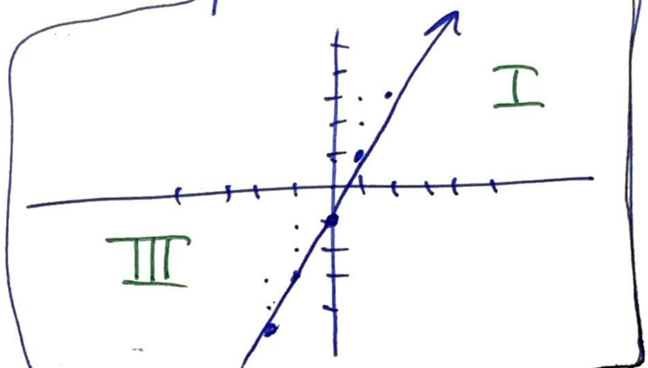
ex $f(x) = 2 \longrightarrow$



Linear functions (a.k.a. lines)

$f(x) = mx + b$

Ex $f(x) = 2x - 1 \xrightarrow{\text{graph}} y = 2x - 1$

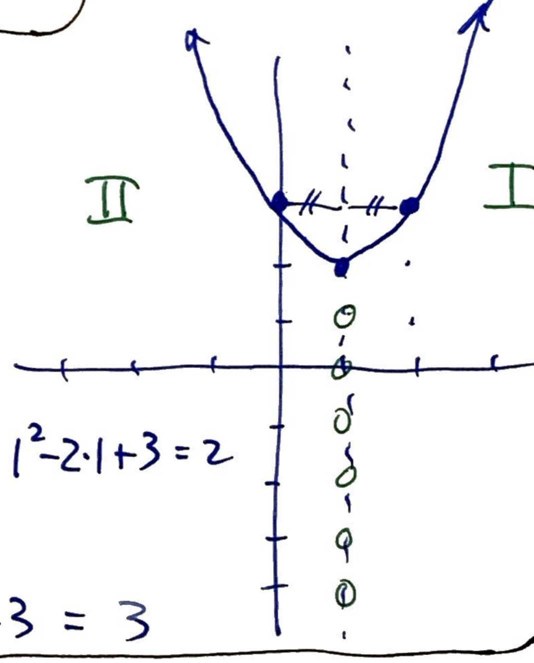


quadratic functions (aka parabola)

$f(x) = ax^2 + bx + c$

Ex $f(x) = x^2 - 2x + 3$

$y = x^2 - 2x + 3$



- L. of Symmetry: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-2)}{2(1)} = 1$

- vertex: $y = 1^2 - 2\cdot 1 + 3 = 2$

- Intercept: @ $x=0$ $y = 0^2 - 2\cdot 0 + 3 = 3$

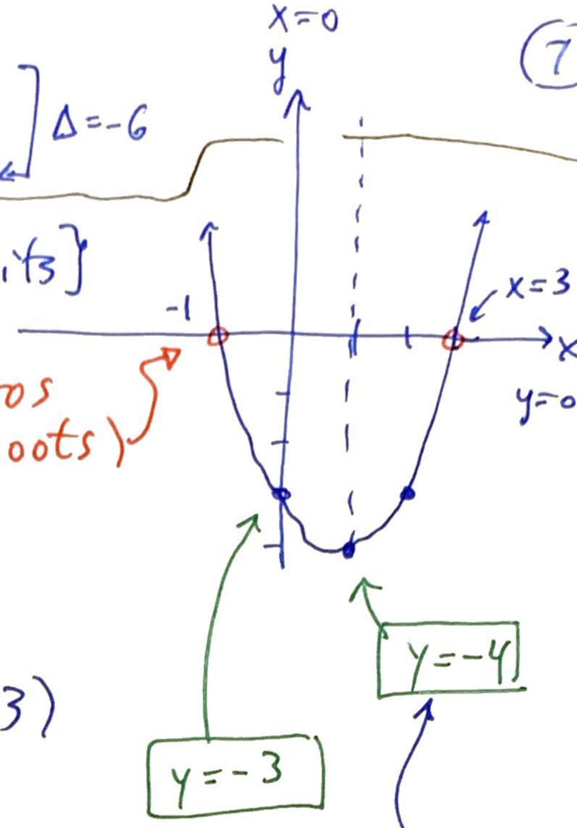
⊛ Quadratic (cont.)

$f(x) = x^2 - 2x + 3$

$\Delta = -6$

Ex $h(x) = x^2 - 2x - 3$

{drop the previous graph 6 units}



• LoS $x = \frac{-b}{2a} = \frac{-(-2)}{2\cdot 1} = \boxed{1}$

zeros (roots)

• roots: set $y = 0$

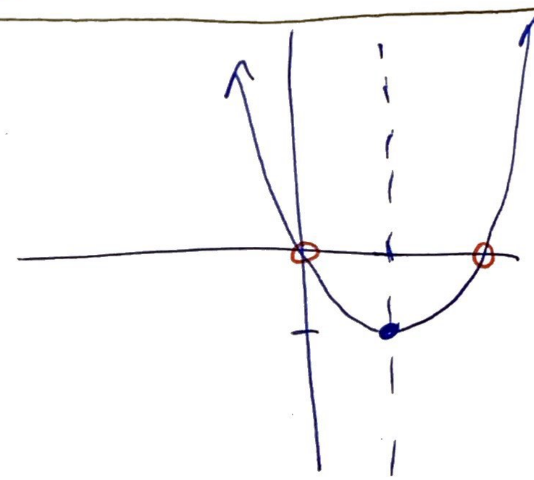
$0 = x^2 - 2x - 3$

$0 = (x+1)(x-3)$

$\boxed{x = -1,\ x = 3}$

• vertex: $y = 1^2 - 2\cdot 1 - 3 = \underline{\underline{-4}}$

Ex $g(x) = x^2 - 2x$



• L.o.S. $x = \frac{-b}{2a} = 1$

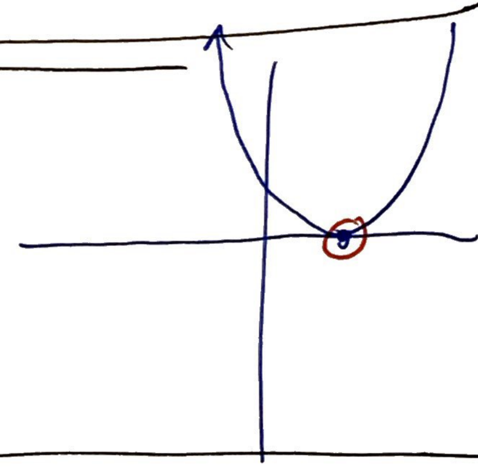
vertex: $y = 1^2 - 2\cdot 1 = -1$

• roots: $y = x^2 - 2x$

$= (x-2)x$

$\boxed{x = 2, 0}$

Ex $F(x) = x^2 - 2x + 1$



boost $g(x)$ up one unit...

A single root at $x = 1$

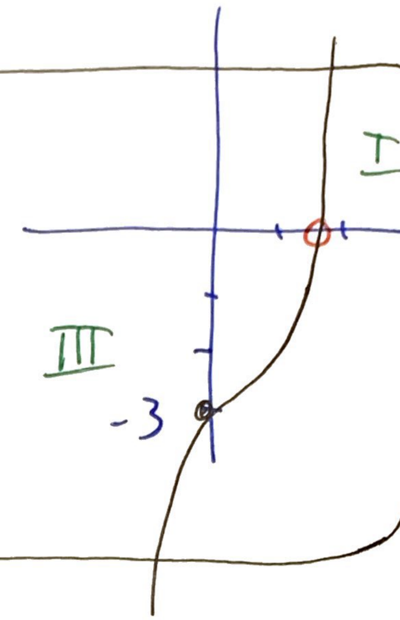
cubic functions

$$\boxed{f(x) = ax^3 + bx^2 + cx + d}$$

Ex $f(x) = x^3 - x^2 + 2x - 3$

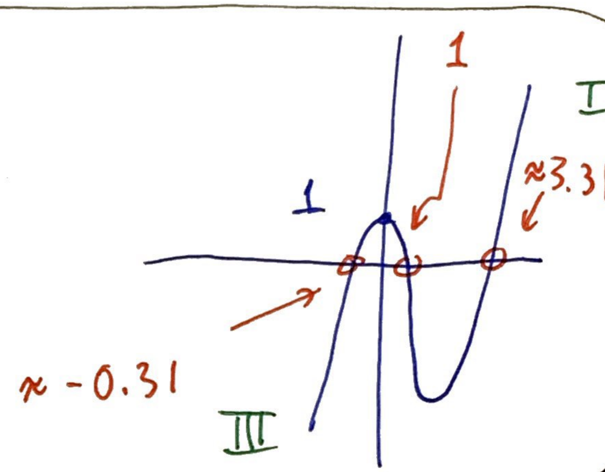
- roots: $\underbrace{0 = x^3 - x^2 + 2x - 3}_{}$

$\left\{\begin{array}{l}\text{one real root}\\ \text{two imaginary}\end{array}\right.$



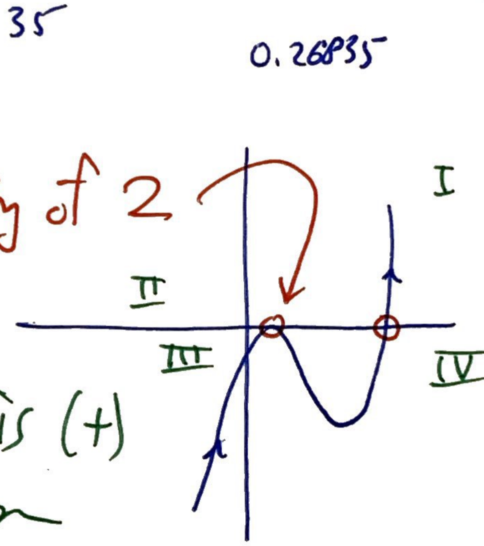
Ex $g(x) = x^3 - 4x^2 + 2x + 1$

three real roots



Ex $h(x) = x^3 - 4x^2 + 2x - 0.26835$

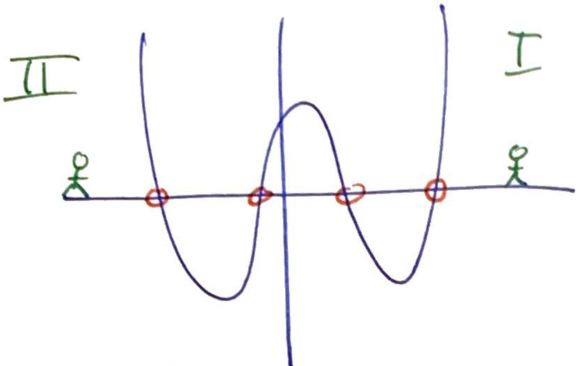
two real roots but one has multiplicity of 2



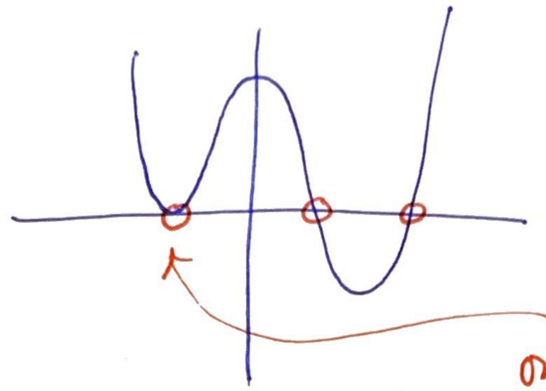
When the leading cubic term is (+) then the cubic travels from Quadrant III to I

* quartic

$$f(x) = ax^{\boxed{4}} + bx^3 + cx^2 + dx + s$$



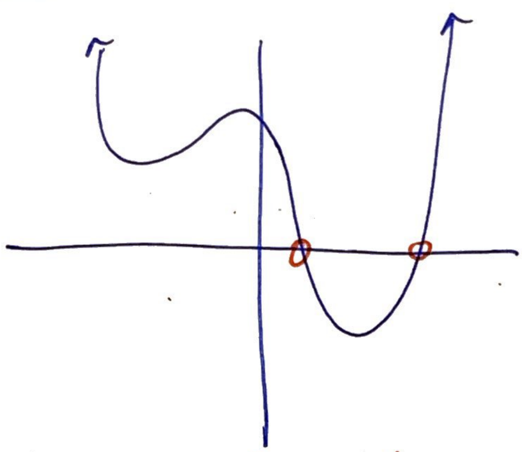
$\boxed{4}$ real roots



4 real roots, one has a multiplicity of two

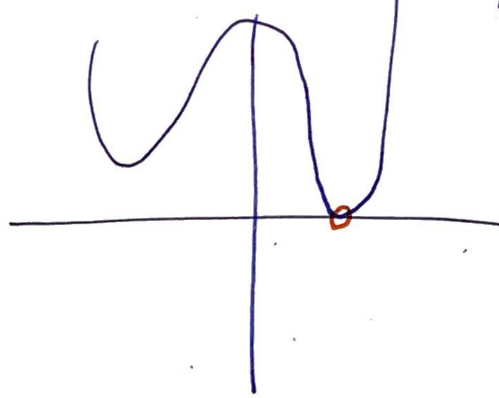
2 has mult=1

$\boxed{4}$



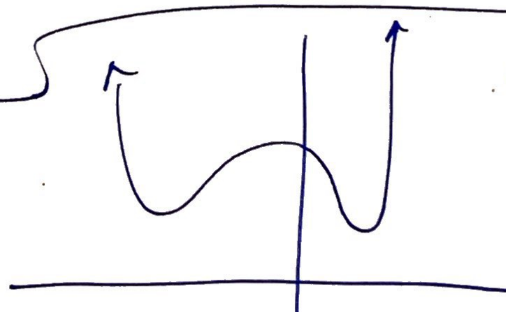
2 real roots
+ 2 imaginary root

$\boxed{4}$



1 real root, mult = 2
2 imag. roots

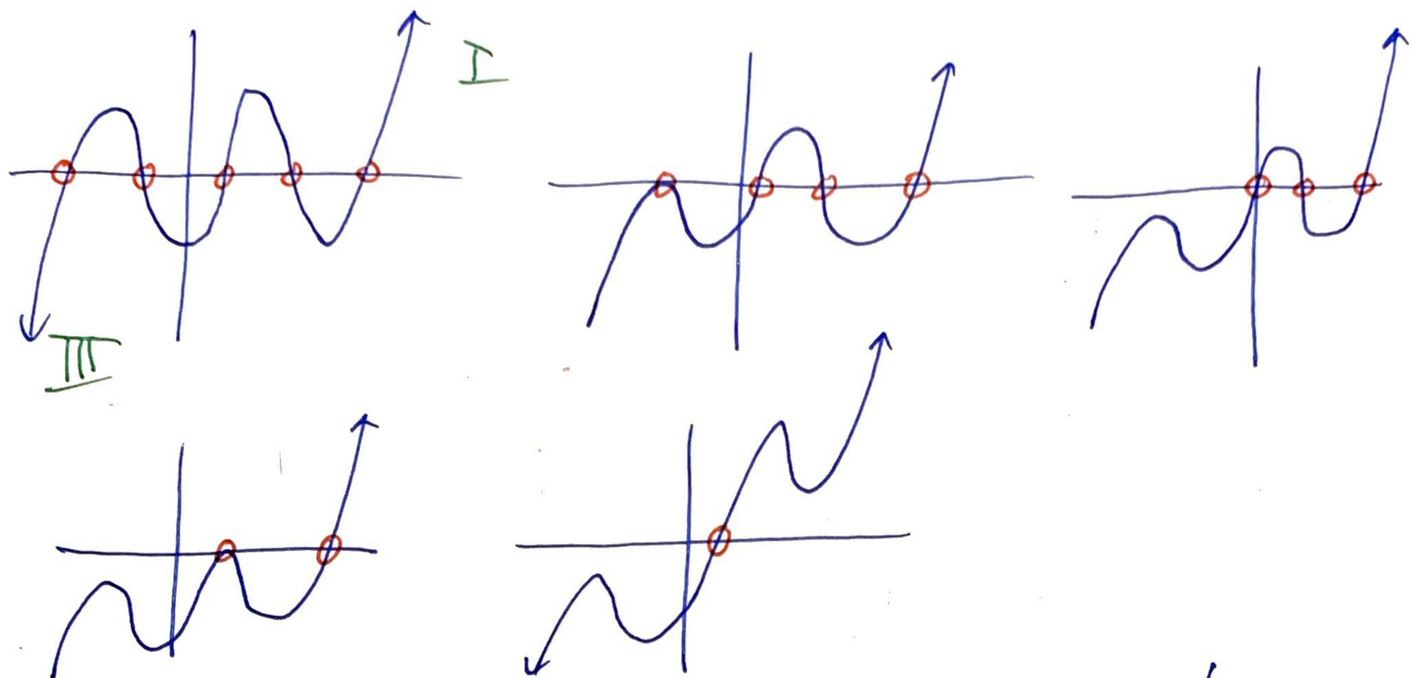
$= \boxed{4}$



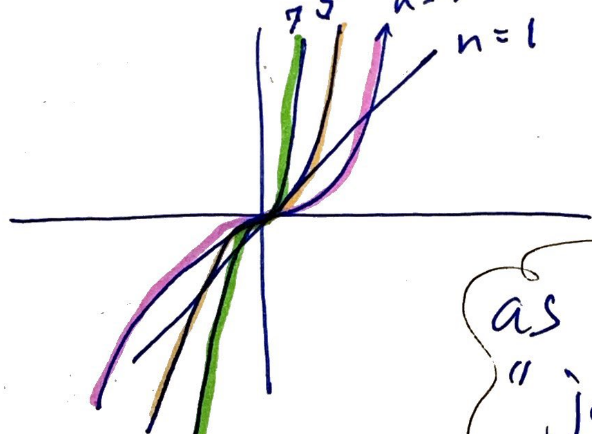
0 real roots

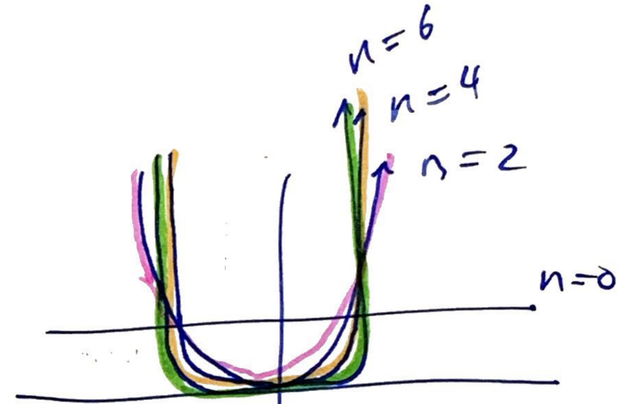
2 sets of imaginary roots that come in conjugate pairs so $2 \times 2 = \boxed{4}$ roots

⊛ quintic $f(x) = ax^{\overset{\text{odd}}{\boxed{5}}} + \dots$



⊛ Just $f(x) = x^n$

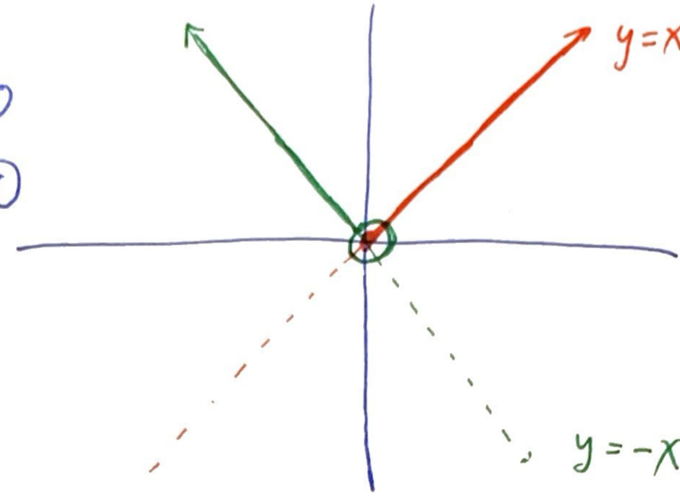




as n increases the "jaw" shape becomes more square.

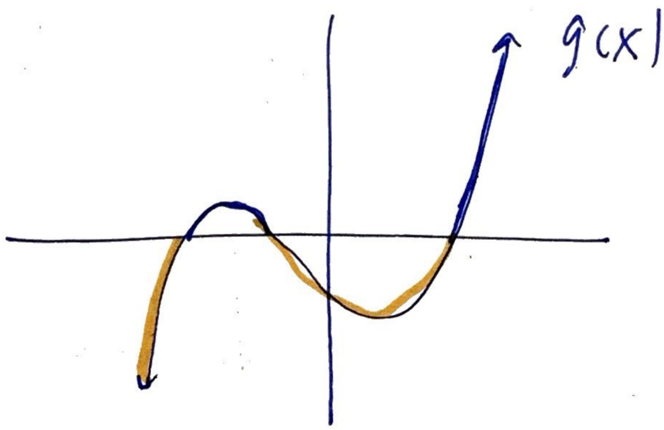
⊛ absolute value

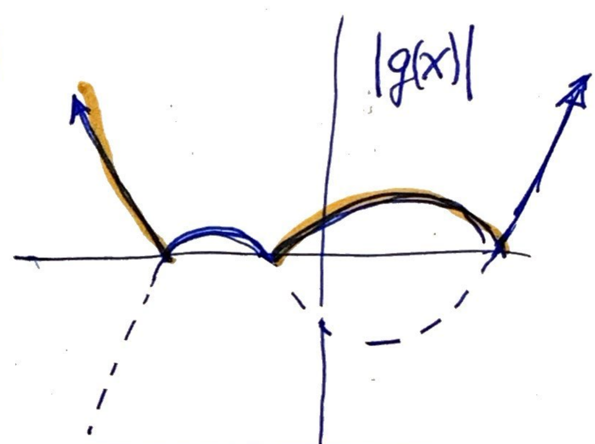
$$f(x) = \begin{cases} \boxed{x} & x \geq 0 \\ \boxed{-x} & x < 0 \end{cases}$$



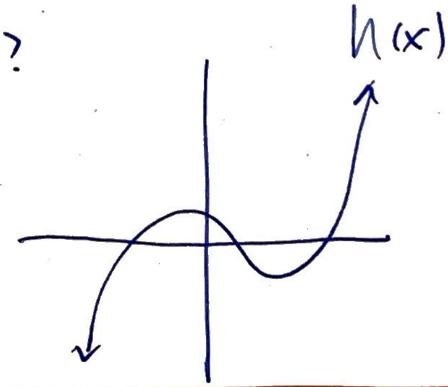
- generically

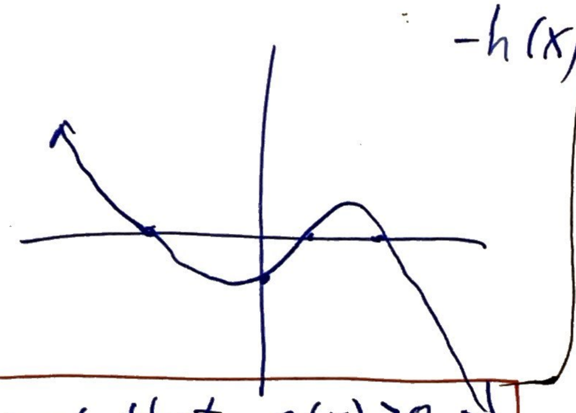
$f(x) = |g(x)|$ then we flip the portion of $g(x)$ under the x-axis up above the x-axis.





why?

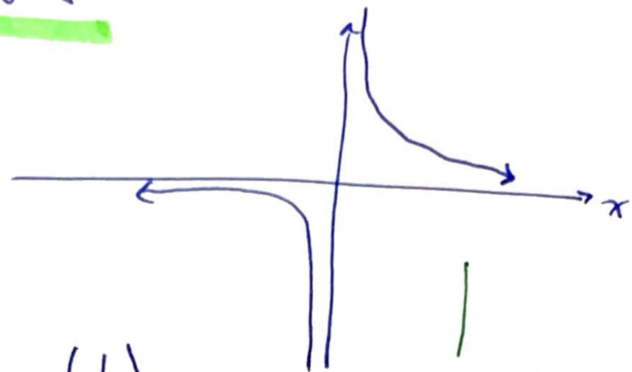




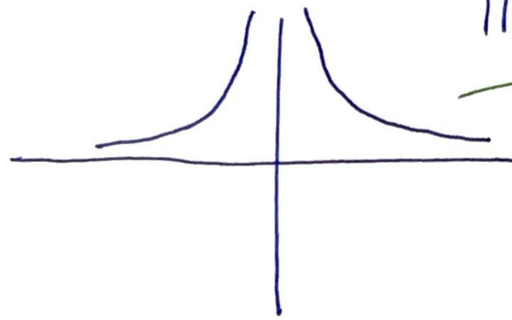
$$f(x) = \begin{cases} g(x) & \text{where } x \text{ is such that } g(x) > 0 \\ -g(x) & \text{where } x \text{ is such that } g(x) < 0 \end{cases}$$

⊛ Reciprocal function

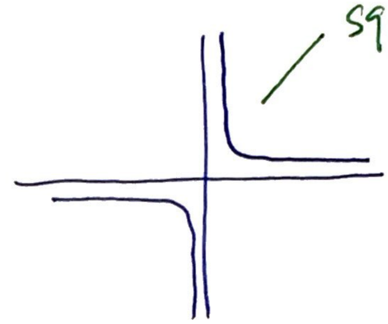
- $f(x) = \frac{1}{x}$



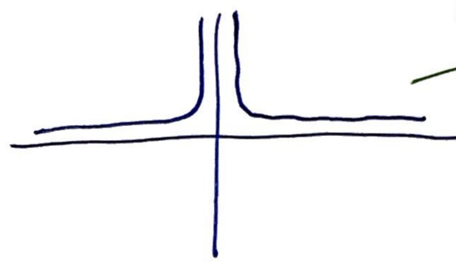
- $g(x) = \frac{1}{x^2}$



- $h(x) = \frac{1}{x^3}$



- $F(x) = \frac{1}{x^4}$



⊛ Naming terms

linear term

$$f(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$$

quintic term

quadric term

cubic term

quadratic term

constant term