16. $x^3y''' + xy' - y = 0$ $\ln x = s$ $\frac{dv}{dx} = \frac{1}{x}$ $2x \frac{d^2y}{dx^2} + x^2 \frac{d^3y}{dx^3} =$

$\frac{dy}{dx} = \frac{dy}{dw} \frac{dv}{dx} = \frac{1}{x}\frac{dy}{dv} = x\frac{dy}{dv} = \frac{dy}{dv}$ $\frac{d^3y}{dv^2} \cdot \frac{dv}{dx} - \frac{dv}{dv^2} \frac{dv}{dx}$

reenshot

$x \frac{d^2y}{dx^2} + \frac{dy}{dv} = \frac{d^2y}{dv^2} \cdot \frac{dv}{dx}$ $2x^2 \frac{d^2y}{dx^2} + x^3 \frac{d^3y}{dv^2} = \frac{d^3y}{dv^3} - \frac{d^2y}{dv^2}$

$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = \frac{d^2y}{dv^2}$

$\frac{x^3 \frac{d^3y}{}}{dx^3} = \frac{d^3y}{dv^3} - \frac{d^2y}{dv^2} - 2\frac{d^2y}{dv^2} + 2\frac{dy}{dv}$

$x^2 \frac{d^2y}{dx^2} = \frac{d^2y}{dv^2} - \frac{dy}{dv}$ $x^3 \frac{d^3y}{dx^2} = \frac{d^3y}{dv^3} - 3\frac{d^2y}{dv^2} + 2\frac{dy}{dv}$ -10

can't follow this

$r^3 - 3r^2 + 3r - 1 = 1$ $\frac{d^3y}{dv^3} - 3\frac{d^2y}{dv^2} + 2\frac{dy}{dv} + \frac{dy}{dv} - y = 0$

$r = 1, 1, 1$

$c_1 e^v + c_2 v e^v + c_3 v^2 e^v$ $c_1 e^{\ln x} + c_2 \ln x e^{\ln x} + c_2 (\ln x)^2 e^{\ln x}$

$y(x) = 4x + c_2 x \ln x + c_3 x (\ln x)^2$

17. $xy^4 + 6y''' = 0$ $x = e^x$ $xD = \theta$ $x^2D^2 = \theta(\theta-1)$ $x^3D^3 = \theta(\theta-1)(\theta-2)$

$x^4y^4 + 6x^3y''' = 0$ $\log x = z$ $x^4D^4 = \theta(\theta-1)(\theta-2)(\theta-3)$

$(x^4D^2 + 6x^3D^3)y = 0$ $\frac{d}{dx} = D$ < we don't use D notation!

$(\theta(\theta-1)(\theta-2)(\theta-3) + 6\theta(\theta-1)(\theta-2))y = 0$ $y = c_1 + c_2e^x + c_3e^{2x} + c_4e^{-3x}$

$(\theta(\theta-1)(\theta-2)((\theta-3) + 6))y = 0$ $y = c_1 + c_2x + c_3x^2 + c_4\frac{1}{x^3}$

$(\theta(\theta-1)(\theta-2)(\theta+3))y = 0$

$m(m-1)(m-2)(m+3) = 0$ $0, 1, 2, -3$

$x^m(25m(m-1)+25m+1)=0$

$x^m(25m^2-25m+25m+1)=0$

$x^m(25m^2+1)=0$

$x^m(5m+i)(5m-i)=0$

$(5m+i)(5m-i)=0$

$m_1=-\frac{i}{5}\quad m_2=\frac{i}{5}$

$y=C_1\cos\left(\frac{1}{5}\ln x\right)+C_2\sin\left(\frac{1}{5}\ln x\right)$

$x^3y'''-6y=0,$

$y'=mx^{(m-1)}\quad y''=m(m-1)x^{(m-2)}$

$y'''=m(m-1)(m-2)\,x^{m-3},$

$x^3(m(m-1)(m-2)(x^{(m-3)}))-6x^m=0$

$x^3x^{m-3}m(m-1)(m-2)-6x^m=0$

$m(m-1)(m-2)x^m-6x^m=0$

$(m^2-m)(m-2)x^m-6x^m=0$

$(m^3-2m^2-m^2+2m-6)x^m=0$

$(m^3-\ldots$

Math 275 - Project 5: Higher-order ODEs - Cauchy-Euler Equations

#1, 5, 9, 15-19, 33, 36

In Problems 1 – 18 solve the given differential equation.

1) $x^2y'' - 2y = 0$

Trial solution: $y = x^m$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$x^2(m(m-1)x^{m-2}) - 2x^m = 0$$

$$x^m \cdot m(m-1) - x^m \cdot 2 = 0$$

$$x^m \underbrace{[m(m-1) - 2]}_{\text{auxilary equation}} = 0$$

$$m(m-1) - 2 = 0$$

$$m^2 - m - 2 = 0$$

$$(m+1)(m-2) = 0$$

$$m = -1, m = 2$$

general solution:

$$\boxed{y = c_1x^{-1} + c_2x^2}$$

C

5) $x^2y'' + xy' + 4y = 0$

Trial solution: $y = x^m$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$x^2(m(m-1)x^{m-2}) + x(mx^{m-1}) + 4x^m = 0$$

$$x^m \cdot m(m-1) + x^m \cdot m + x^m \cdot 4 = 0$$

$u_1' = \frac{-1}{5} x^4 \qquad u_2' = \frac{1}{5} x^{-1}$

$u_1 = \int \frac{-1}{5} x^4 \qquad u_2 = \int \frac{1}{5} x^{-1}$

$= \frac{-1}{5} \cdot \frac{1}{5} x^5 \qquad = \frac{1}{5} \ln x$

$= \frac{-1}{25} x^5$

How are u1 and u2 used? -2

$y_p = \frac{-1}{25} x^5 + \frac{1}{5} \ln x \,(x^5)$

$y = y_c + y_p$

$$\boxed{y = c_1 + c_2 x^5 - \frac{1}{25} x^5 + \frac{1}{5} \ln x \,(x^5)}$$

In Problems 31–36 use the substitution $x = e^t$ to transform the given Cauchy-Euler equation to a differential equation with constant coefficients. Solve the original equation by solving the new equation using the procedures in Sections 4.3–4.5.

33) $x^2y'' + 10xy' + 8y = x^2$

$x = e^t$

$t = \ln x$

$\frac{dt}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{1}{x} \cdot \frac{dy}{dt}$

$\frac{dy^2}{dx^2} = \frac{1}{x} \cdot \frac{d}{dx}\left(\frac{dy}{dt}\right) - \frac{1}{x^2}\left(\frac{dy}{dt}\right)$

$= \frac{1}{x}\left(\frac{dy^2}{dt^2}\right)\left(\frac{1}{x}\right) - \frac{1}{x^2}\left(\frac{dy}{dt}\right)$