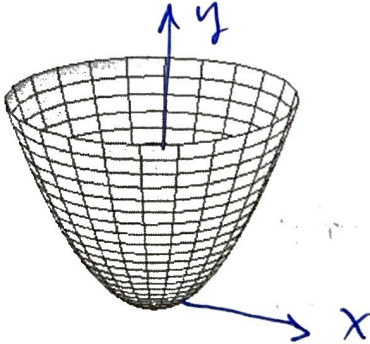


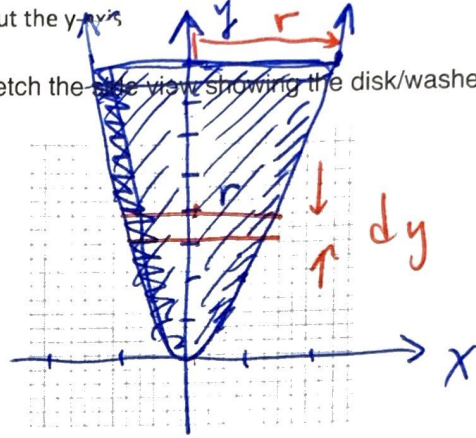
Show work for full credit. Go for the most point. Don't get bogged down on any one problem.

1. [5.2 Volumes] (10 pts). Use disk/washers to determine the volume of the solid obtained by rotating the region bounded by $y = 2x^2$, $y = 8$ and the y-axis about the y-axis

(i) Sketch the x & y axis on this 3-D view:



(ii) Sketch the side view showing the disk/washer:



(iii) What will the thickness be? dx or dy ? (Circle one) Indicate this on the side view sketch above.

(iv) What will the radius and area of the disk/washer be in terms of the variable selected in (iii)?

radius: $r = x$

$r = \sqrt{y/2}$

$y = 2x^2$
 $x = \sqrt{\frac{y}{2}}$

area = $\pi r^2 = \pi x^2$

$= \pi y/2$

(v) What will the volume of the disk/washer be? Volume = area * thickness

volume = $\pi y/2 \cdot dy$

(vi) What are the variable limits in terms of the variable selected in (iii)? circle it below

a: from x or $y = 0$ to b: x or $y = 8$

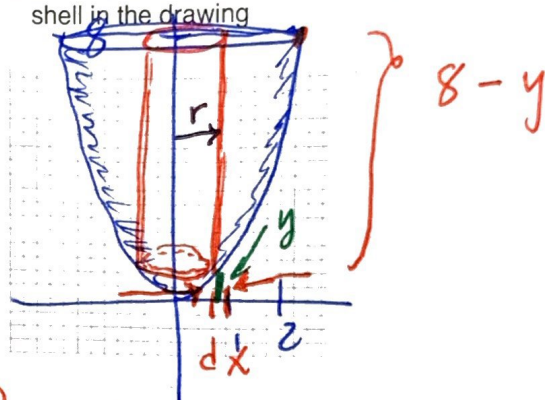
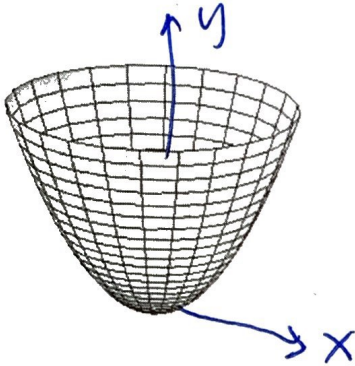
(v) Set the integral up but do not solve it

$$V = \int_{a=0}^{b=8} \frac{\pi y}{2} dy$$

10

2. [5.3 Volumes by Shells] (10 pts) Use cylindrical shells to determine the volume of the solid obtained by rotating the region bounded by $y = 2x^2$, $y = 8$ and the y -axis about the y -axis (yes, same as prob 1)

- (i) Sketch the x & y axii on this 3-D view: (ii) Sketch the side-view. Place a thin cylindrical shell in the drawing



(iii) What will the thickness of the shell be? dx or dy ? (Circle one) Indicate this on the side-view sketch above.

(iv) What will the radius, height and sidewall area of the shell be in terms of the variable selected in (iii)?

radius: $r = x$

height: $h = 8 - y = 8 - 2x^2$

sidewall area = $2\pi r h = 2\pi x (8 - 2x^2)$

(v) What will the volume of the shell be? Volume = shell sidewall area * thickness

volume = $2\pi x (8 - 2x^2) \cdot dx$

(vi) What are the variable limits in terms of the variable selected in (iii)? circle it below

a: from x or $y = 0$ to b: x or $y = 2$

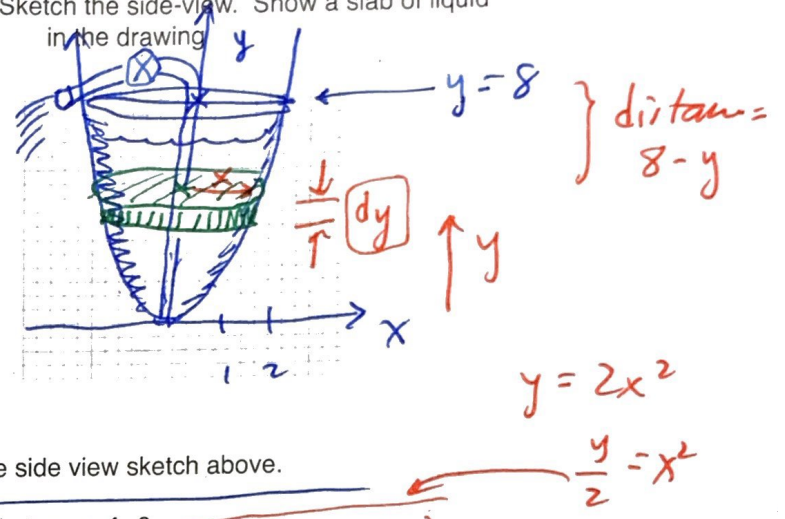
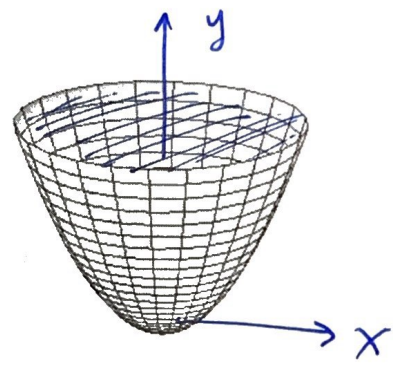
(v) Set the integral up but do not solve it

$$V = \int_{a=0}^{b=2} 2\pi x (8 - 2x^2) dx$$

10)

3. [5.4 Work] (10 pts) Determine the work performed by a pump used to drain the liquid contents of a tank that was formed by rotating the region bounded by $y = 2x^2$, $y = 8$ and the y -axis about the y -axis

- (i) Sketch the x & y axii on this 3-D view: (ii) Sketch the side-view. Show a slab of liquid in the drawing



(iii) Let the thickness be 'dy' Indicate this on the side view sketch above.

(iv) What will the radius and area of the slab be in terms of 'y'?

radius: $r = x$, now convert 'x' to a function of 'y': $r = \sqrt{\frac{y}{2}}$

area = $\pi r^2 = \pi \left(\frac{\sqrt{y}}{\sqrt{2}}\right)^2 = \frac{\pi y}{2}$

(v) What will the volume and weight of the slab be?

-> volume = area * thickness, but thickness is 'dy', so just re-write the area in the blank:

volume = $\frac{\pi y}{2} \cdot dy$

-> weight = $\rho_0 Vg$ {use $g = 10$ and $\rho_0 = 1000$ }

weight = $(1000) \frac{\pi y}{2} (10) \cdot dy = 5000 \pi y \cdot dy$

(vi) How high will the slab be lifted if the exit is just above the 8 meter rim?

distance from slab height-to-exit: $H = 8 - y$

(vii) What are the integral limits if we drain just over the top of the tank?

a: from $y = 0$ to b: $y = 8$

(v) Set the integral up but do not solve it : $W = \text{weight} \cdot \text{distance}$

$W = \int_{a=0}^{b=8} 5000 \pi y (8-y) dy$

10