(CHAPT 11A)

11.2 Intro

- Def: $\{s_n\} \to S$ then $\sum a_n$ is conv.
- Geometric series: $\sum ar^{n-1} = \frac{a}{1-r}$, $|r|<1$
- Thm: If $\sum a_n \to$ conv, $\Rightarrow a_n \to 0$
- Thm: $\{a_n\} \not\to 0 \Rightarrow \sum a_n$ is divergence

11.3 Integral Test

- Integral Test: if $\int_N^\infty f$ is $\begin{Bmatrix}\text{conv}\\ \text{div}\end{Bmatrix} \Rightarrow \sum_N^\infty a_n$ is $\begin{Bmatrix}\text{conv}\\ \text{div}\end{Bmatrix}$ [$f>0$, decr.]
- p-series: $\sum_1^\infty \frac{1}{n^p}$ conv for $p>1$
- Remainder: $\int_{n+1}^\infty f \le R_n \le \int_n^\infty f$
- Estimation: $s_n + \int_{n+1}^\infty f \le s \le \int_n^\infty f + s_n$

11.4 Comparison Tests

- Comp Test: $\begin{cases}\sum b_n \text{ is conv and } a_n \le b_n, \forall n, \text{ then } \sum a_n \text{ is conv.}\\ \sum b_n \text{ is div and } a_n \ge b_n, \forall n, \text{ then } \sum a_n \text{ is div.}\end{cases}$ a_n, b_n positive terms
- Limit Comparison: If $\sum a_n$, $\sum b_n$ are positive termed series and If $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = c$, $c>0$, $\Rightarrow$ both series conv or div.

Test:

- If $a_n \le b_n, \forall n$, then $R_n \le T_n$, remainders for $\sum_1^N a$ & $\sum_1^N b$ accordingly.

11.5 Alternating Series: let $a_n = (-1)^{n-1} b_n$

- Alt. Series Test: If $b_n \to 0$ and if $b_{n+1} \le b_n, \forall n$, then $\sum (-1)^{n-1} b_n \to$ conv
- Est. If $s = \sum (-1)^{n-1} b_n$ and $0 \le b_{n+1} \le b_n$ and $b_n \to 0$ then $|R_n| \le b_{n+1}$

11.6 Abs. Conv:

- Def: $\sum a_n$ is absolutely conv if $\sum |a_n|$ is conv.
- Def: $\sum a_n$ is conditionally conv if $\sum a_n$ conv's but $\sum |a_n|$ div.
- Ratio Test: $\left|\frac{a_{n+1}}{a_n}\right| \to L \begin{Bmatrix}<\\ >\\ =\end{Bmatrix} 1 \to \begin{Bmatrix}\text{abs. conv}\\ \text{div}\\ \text{inconclusive}\end{Bmatrix}$
- Root Test: $\sqrt[n]{|a_n|} \to L \begin{Bmatrix}<\\ >\\ =\end{Bmatrix} 1 \to \begin{Bmatrix}\text{abs. conv}\\ \text{div}\\ \text{inconclusive}\end{Bmatrix}$

Classify via Form:

1. $\sum \frac{1}{n^p}$ p-series $p>1$
2. $\sum ar^{n-1}$ geom $|r|<1$
3. If like above then compare
4. $a_n \not\to 0$ div
5. Alt Series? $b_{n+1} \le b_n$
6. n!? $\left|\frac{a_{n+1}}{a_n}\right| \to L$ <, >, = 1
7. $(b_n)^n$? $\sqrt[n]{|a_n|} \to L$ >, <, = 1
8. $\int_N^\infty f \to \begin{matrix}\text{div}\\ \text{conv.}\end{matrix} \Rightarrow \sum \to \begin{matrix}\text{div}\\ \text{conv}\end{matrix}$