Calc II Cheat Sheets

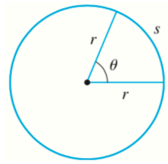
Angle Measurement

π radians $= 180°$

$1° = \frac{\pi}{180}$ rad $\quad$ 1 rad $= \frac{180°}{\pi}$

$s = r\theta$

(θ in radians)

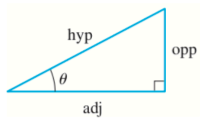


Right Angle Trigonometry

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\quad$ $\csc\theta = \frac{\text{hyp}}{\text{opp}}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\quad$ $\sec\theta = \frac{\text{hyp}}{\text{adj}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}}$ $\quad$ $\cot\theta = \frac{\text{adj}}{\text{opp}}$

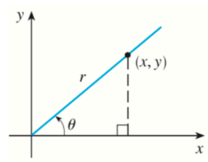


Trigonometric Functions

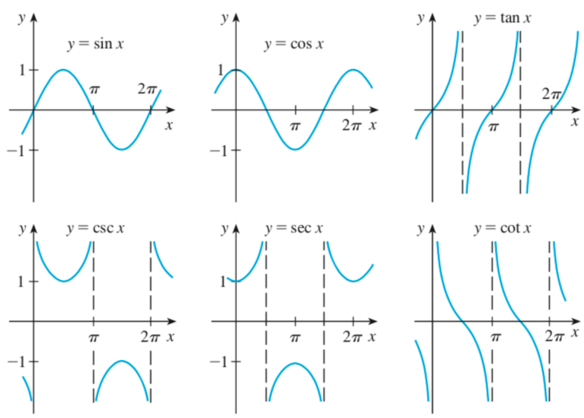
$\sin\theta = \frac{y}{r}$ $\quad$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\quad$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\quad$ $\cot\theta = \frac{x}{y}$



Graphs of Trigonometric Functions



Trigonometric Functions of Important Angles

θ	radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
$0°$	0	0	1	0
$30°$	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
$45°$	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$60°$	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$90°$	$\pi/2$	1	0	—

Fundamental Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\quad$ $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\cot\theta = \frac{1}{\tan\theta}$ $\quad$ $\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$ $\quad$ $1 + \cot^2\theta = \csc^2\theta$

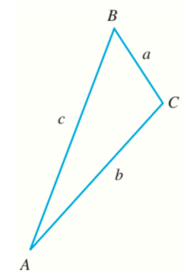
$\sin(-\theta) = -\sin\theta$ $\quad$ $\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$ $\quad$ $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\quad$ $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



The Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Addition and Subtraction Formulas

$\sin(x + y) = \sin x\cos y + \cos x\sin y$

$\sin(x - y) = \sin x\cos y - \cos x\sin y$

$\cos(x + y) = \cos x\cos y - \sin x\sin y$

$\cos(x - y) = \cos x\cos y + \sin x\sin y$

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$

$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$

Double-Angle Formulas

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

Half-Angle Formulas

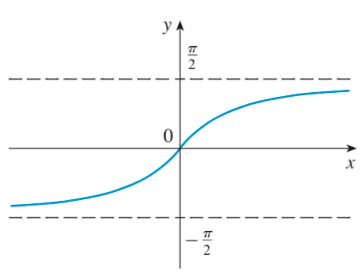
$\sin^2 x = \frac{1 - \cos 2x}{2}$ $\quad$ $\cos^2 x = \frac{1 + \cos 2x}{2}$

Inverse Trigonometric Functions

$\arcsin x = \sin^{-1}x = y \iff \sin y = x$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$\arccos x = \cos^{-1}x = y \iff \cos y = x$ and $0 \leq y \leq \pi$

$\arctan x = \tan^{-1}x = y \iff \tan y = x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$



$y = \tan^{-1}x = \arctan x$

$\lim_{x\to-\infty} \tan^{-1}x = -\frac{\pi}{2}$

$\lim_{x\to\infty} \tan^{-1}x = \frac{\pi}{2}$

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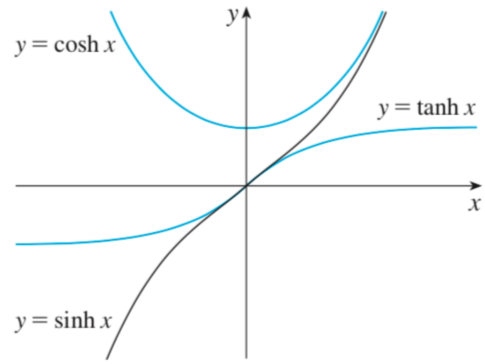
Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{\cosh x}{\sinh x}$$



Inverse Hyperbolic Functions

$$y = \sinh^{-1}x \iff \sinh y = x$$

$$y = \cosh^{-1}x \iff \cosh y = x \text{ and } y \geqslant 0$$

$$y = \tanh^{-1}x \iff \tanh y = x$$

$$\sinh^{-1}x = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$\cosh^{-1}x = \ln\left(x + \sqrt{x^2 - 1}\right)$$

$$\tanh^{-1}x = \tfrac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right)$$

Arithmetic Operations

$$a(b + c) = ab + ac \qquad \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b} \qquad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Exponents and Radicals

$$x^m x^n = x^{m+n} \qquad \frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{mn} \qquad x^{-n} = \frac{1}{x^n}$$

$$(xy)^n = x^n y^n \qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{1/n} = \sqrt[n]{x} \qquad x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

$$\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y} \qquad \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

Factoring Special Polynomials

$$x^2 - y^2 = (x + y)(x - y)$$

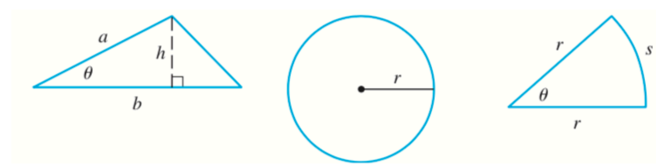
$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

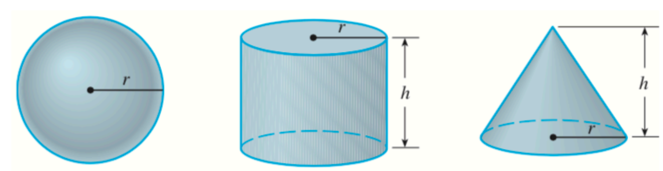
Geometric Formulas

Formulas for area A, circumference C, and volume V:

Triangle	Circle	Sector of Circle
$A = \frac{1}{2}bh$	$A = \pi r^2$	$A = \frac{1}{2}r^2\theta$
$= \frac{1}{2}ab \sin\theta$	$C = 2\pi r$	$s = r\theta$ (θ in radians)



Sphere	Cylinder	Cone
$V = \frac{4}{3}\pi r^3$	$V = \pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$
$A = 4\pi r^2$		$A = \pi r\sqrt{r^2 + h^2}$



Calc II Cheat Sheets

General Formulas

1. $\frac{d}{dx}(c) = 0$

2. $\frac{d}{dx}[cf(x)] = cf'(x)$

3. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

4. $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

5. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (Product Rule)

6. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ (Quotient I

7. $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (Chain Rule)

8. $\frac{d}{dx}(x^n) = nx^{n-1}$ (Power Rule)

Exponential and Logarithmic Functions

9. $\frac{d}{dx}(e^x) = e^x$

10. $\frac{d}{dx}(b^x) = b^x \ln b$

11. $\frac{d}{dx}\ln|x| = \frac{1}{x}$

12. $\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$

Trigonometric Functions

13. $\frac{d}{dx}(\sin x) = \cos x$

14. $\frac{d}{dx}(\cos x) = -\sin x$

15. $\frac{d}{dx}(\tan x) = \sec^2 x$

16. $\frac{d}{dx}(\csc x) = -\csc x \cot x$

17. $\frac{d}{dx}(\sec x) = \sec x \tan x$

18. $\frac{d}{dx}(\cot x) = -\csc^2 x$

Inverse Trigonometric Functions

19. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1 - x^2}}$

20. $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1 - x^2}}$

21. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1 + x^2}$

22. $\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2 - 1}}$

23. $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2 - 1}}$

24. $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1 + x^2}$

Hyperbolic Functions

25. $\frac{d}{dx}(\sinh x) = \cosh x$

26. $\frac{d}{dx}(\cosh x) = \sinh x$

27. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

28. $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

29. $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

30. $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

Inverse Hyperbolic Functions

31. $\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1 + x^2}}$

32. $\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2 - 1}}$

33. $\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1 - x^2}$

34. $\frac{d}{dx}(\operatorname{csch}^{-1}x) = -\frac{1}{|x|\sqrt{x^2 + 1}}$

35. $\frac{d}{dx}(\operatorname{sech}^{-1}x) = -\frac{1}{x\sqrt{1 - x^2}}$

36. $\frac{d}{dx}(\coth^{-1}x) = \frac{1}{1 - x^2}$

Basic Forms

1. $\int u\,dv = uv - \int v\,du$

2. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$

3. $\int \frac{du}{u} = \ln|u| + C$

4. $\int e^u\,du = e^u + C$

5. $\int b^u\,du = \frac{b^u}{\ln b} + C$

6. $\int \sin u\,du = -\cos u + C$

7. $\int \cos u\,du = \sin u + C$

8. $\int \sec^2 u\,du = \tan u + C$

9. $\int \csc^2 u\,du = -\cot u + C$

10. $\int \sec u \tan u\,du = \sec u + C$

11. $\int \csc u \cot u\,du = -\csc u + C$

12. $\int \tan u\,du = \ln|\sec u| + C$

13. $\int \cot u\,du = \ln|\sin u| + C$

14. $\int \sec u\,du = \ln|\sec u + \tan u| + C$

15. $\int \csc u\,du = \ln|\csc u - \cot u| + C$

16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a} + C, \quad a > 0$

17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

18. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$

19. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a}\ln\left|\frac{u + a}{u - a}\right| + C$

20. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a}\ln\left|\frac{u - a}{u + a}\right| + C$

Forms Involving $\sqrt{a^2 + u^2}$, $a > 0$

21. $\int \sqrt{a^2 + u^2}\,du = \frac{u}{2}\sqrt{a^2 + u^2} + \frac{a^2}{2}\ln\left(u + \sqrt{a^2 + u^2}\right) + C$

22. $\int u^2\sqrt{a^2 + u^2}\,du = \frac{u}{8}(a^2 + 2u^2)\sqrt{a^2 + u^2} - \frac{a^4}{8}\ln\left(u + \sqrt{a^2 + u^2}\right) + C$

23. $\int \frac{\sqrt{a^2 + u^2}}{u}\,du = \sqrt{a^2 + u^2} - a\ln\left|\frac{a + \sqrt{a^2 + u^2}}{u}\right| + C$

24. $\int \frac{\sqrt{a^2 + u^2}}{u^2}\,du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln\left(u + \sqrt{a^2 + u^2}\right) + C$

25. $\int \frac{du}{\sqrt{a^2 + u^2}} = \ln\left(u + \sqrt{a^2 + u^2}\right) + C$

30. $\int \sqrt{a^2 - u^2}\,du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$

39. $\int \sqrt{u^2 - a^2}\,du = \frac{u}{2}\sqrt{u^2 - a^2} - \frac{a^2}{2}\ln\left|u + \sqrt{u^2 - a^2}\right| + C$

Calc II Cheat Sheets

Midpoint Rule:

$$I \approx M_n = \Delta x\,[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)]$$

where $\bar{x}_i$ is the midpoint of $[x_{i-1}, x_i]$.

Trapezoidal Rule:

$$I \approx T_n = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

Simpson's Rule:

$$I \approx S_n = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

where n is even.

Write an expression for the length of a smooth curve given by $y = f(x)$, $a \leqslant x \leqslant b$.

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

What if x is given as a function of y?

If $x = g(y)$, $c \leqslant y \leqslant d$, then $L = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy$.

Write an expression for the surface area of the surface obtained by rotating the curve $y = f(x)$, $a \leqslant x \leqslant b$, about the x-axis.

$$S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\, dx$$

What if x is given as a function of y?

If $x = g(y)$, $c \leqslant y \leqslant d$, then $S = \int_c^d 2\pi y\sqrt{1 + [g'(y)]^2}\, dy$.

What if the curve is rotated about the y-axis?

$$S = \int_a^b 2\pi x\sqrt{1 + [f'(x)]^2}\, dx$$

or

$$S = \int_c^d 2\pi g(y)\sqrt{1 + [g'(y)]^2}\, dy$$

How do you find the slope of a tangent line to a polar curve?

$$\frac{dy}{dx} = \frac{\dfrac{dy}{d\theta}}{\dfrac{dx}{d\theta}} = \frac{\dfrac{d}{d\theta}(y)}{\dfrac{d}{d\theta}(x)} = \frac{\dfrac{d}{d\theta}(r\sin\theta)}{\dfrac{d}{d\theta}(r\cos\theta)}$$

$$= \frac{\left(\dfrac{dr}{d\theta}\right)\sin\theta + r\cos\theta}{\left(\dfrac{dr}{d\theta}\right)\cos\theta - r\sin\theta} \quad \text{where } r = f(\theta)$$

Suppose $|f''(x)| \leqslant K$ and $|f^{(4)}(x)| \leqslant L$ for $a \leqslant x \leqslant b$. The errors in the Midpoint, Trapezoidal, and Simpson's Rules are given by, respectively,

$$|E_M| \leqslant \frac{K(b-a)^3}{24n^2} \qquad |E_T| \leqslant \frac{K(b-a)^3}{12n^2}$$

$$|E_S| \leqslant \frac{L(b-a)^5}{180n^4}$$

How do you find the slope of a tangent to a parametric curve?

You can find dy/dx as a function of t by calculating

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \text{if } dx/dt \neq 0$$

How do you find the area under a parametric curve?

If the curve is traced out once by the parametric equations $x = f(t)$, $y = g(t)$, $\alpha \leqslant t \leqslant \beta$, then the area is

$$A = \int_a^b y\, dx = \int_\alpha^\beta g(t)f'(t)\, dt$$

[or $\int_\beta^\alpha g(t)f'(t)\, dt$ if the leftmost point is $(f(\beta), g(\beta))$ rather than $(f(\alpha), g(\alpha))$].

ite an expression for each of the following:

The length of a parametric curve

If the curve is traced out once by the parametric equations $x = f(t)$, $y = g(t)$, $\alpha \leqslant t \leqslant \beta$, then the length is

$$L = \int_\alpha^\beta \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt = \int_\alpha^\beta \sqrt{[f'(t)]^2 + [g'(t)]^2}\, dt$$

The area of the surface obtained by rotating a parametric curve about the x-axis

$$S = \int_\alpha^\beta 2\pi y\sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt = \int_\alpha^\beta 2\pi g(t)\sqrt{[f'(t)]^2 + [g'(t)]^2}\, dt$$

How do you find the area of a region bounded by a polar curve?

$$A = \int_a^b \tfrac{1}{2}r^2\, d\theta = \int_a^b \tfrac{1}{2}[f(\theta)]^2\, d\theta$$

How do you find the length of a polar curve?

$$L = \int_a^b \sqrt{(dx/d\theta)^2 + (dy/d\theta)^2}\, d\theta = \int_a^b \sqrt{r^2 + (dr/d\theta)^2}\, d\theta = \int_a^b \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\, d\theta$$

(CHAPT 11A)

11.2 Intro

- Def: $\{s_n\} \to s$ then $\sum a_n$ is <u>conv</u>.
- Geometric series: $\sum ar^{n-1} = \frac{a}{1-r}$, $|r| < 1$
- Thm: If $\sum a_n \to$ conv, $\Rightarrow a_n \to 0$
- Thm: $\{a_n\} \not\to 0 \Rightarrow \sum a_n$ is divergence

11.3 Integral Test

- Integral Test: if $\int_N^\infty f$ is $\begin{Bmatrix} \text{conv} \\ \text{div} \end{Bmatrix} \Rightarrow \sum_N^\infty a_n$ is $\begin{Bmatrix} \text{conv} \\ \text{div} \end{Bmatrix}$ [$f > 0$, decr.]
- p-series: $\sum_1^\infty \frac{1}{n^p}$ conv for $p > 1$
- Remainder: $\int_{n+1}^\infty f \leq R_n \leq \int_n^\infty f$
- Estimation: $s_n + \int_{n+1}^\infty f \leq s \leq \int_n^\infty f + s_n$

Classify via Form:

1. $\sum \frac{1}{n^p}$ p-series $p > 1$
2. $\sum ar^{n-1}$ geom $|r| < 1$
3. If like above then compare
4. $a_n \not\to 0$ div
5. Alt Series? $b_{n+1} \leq b_n$
6. $n!$? $\left|\frac{a_{n+1}}{a_n}\right| \to L <, >, = 1$
7. $(b_n)^n$? $\sqrt[n]{|a_n|} \to L >, <, = 1$
8. $\int_N^\infty f \to \begin{matrix} \text{div} \\ \text{conv} \end{matrix} \Rightarrow \sum \to \begin{matrix} \text{div} \\ \text{conv} \end{matrix}$

11.4 Comparison Tests

- Comp Test: $\begin{cases} \sum b_n \text{ is conv and } a_n \leq b_n, \forall n, \text{ then } \sum a_n \text{ is conv.} \\ \sum b_n \text{ is div and } a_n \geq b_n, \forall n, \text{ then } \sum a_n \text{ is div.} \end{cases}$ a_n, b_n positive terms
- Limit Comparison: If $\sum a_n$, $\sum b_n$ are positive termed series and If $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = c$, $c > 0$, $\Rightarrow$ both series conv or div.

Test:

- If $a_n \leq b_n, \forall n$, then $R_n \leq T_n$, remainders for $\sum_1^N a$ & $\sum_1^N b$ accordingly.

11.5 Alternating Series: let $a_n = (-1)^{n-1} b_n$

- Alt. Series Test: If $b_n \to 0$ and if $b_{n+1} \leq b_n, \forall n$, then $\sum (-1)^{n-1} b_n \to$ conv
- Est. If $s = \sum (-1)^{n-1} b_n$ and $0 \leq b_{n+1} \leq b_n$ and $b_n \to 0$ then $|R_n| \leq b_{n+1}$

11.6 Abs. Conv:

- Def: $\sum a_n$ is <u>absolutely conv</u> if $\sum |a_n|$ is conv.
- Def: $\sum a_n$ is <u>conditionally conv</u> if $\sum a_n$ conv's but $\sum |a_n|$ div.
- Ratio Test: $\left|\frac{a_{n+1}}{a_n}\right| \to L \begin{Bmatrix} < \\ > \\ = \end{Bmatrix} 1 \to \begin{Bmatrix} \text{abs. conv} \\ \text{div} \\ \text{inconclusive} \end{Bmatrix}$
- Root Test: $\sqrt[n]{|a_n|} \to L \begin{Bmatrix} < \\ > \\ = \end{Bmatrix} 1 \to \begin{Bmatrix} \text{abs. conv} \\ \text{div} \\ \text{conclusive} \end{Bmatrix}$