§7.4 Integration of Rational Functions using Partial Fractions

1. Consider a **PROPER** rational function

$$f(x) = \frac{P(x)}{Q(x)} \quad , \quad \deg P < \deg Q$$

The degree of P should be less than the degree of Q.

where P and Q are polynomials.

Recall, the degree of a polynomial is the highest power in the polynomial, so for
$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$
we have $\deg P = n$.

2. If the integrand is an **IMPROPER RATIONAL FUNCTION:**

$$\deg P \geq \deg Q$$

first apply long division to obtain a *proper* rational function:

$$f(x) = S(x) + \frac{R(x)}{Q(x)} \quad , \quad \deg R < \deg Q$$

The degree of R should be less than the degree of Q.

3. Factor the denominator and rewrite the rational function in terms of its factors.
   a) For each factor of $Q(x)$ that has the form $(ax+b)^m$, write terms:

$$\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \ldots + \frac{A_m}{(ax+b)^m}$$

   b) For each factor of $Q(x)$ that is an irreducible quadratic $(ax^2+bx+c)^n$, write terms:

$$\frac{A_1 x + B_1}{ax^2+bx+c} + \frac{A_2 x + B_2}{(ax^2+bx+c)^2} + \ldots + \frac{A_n x + B_n}{(ax^2+bx+c)^n}$$

4. Multiply the expression by the least common denominator, then solve for the unknown coefficients.