§7.2 Trigonometric Integrals

$\int \sin^m x \, \cos^n x \, dx$

Odd power of cosine: $n = 2k+1$

Save one cosine factor and use $\boxed{\cos^2 x = 1 - \sin^2 x}$ to express other cosine factors in terms of sine.

$$\int \sin^m x \cos^{2k+1} x \, dx = \int \sin^m x \left(\cos^2 x\right)^k \cos x \, dx = \int \sin^m x \left(1 - \sin^2 x\right)^k \cos x \, dx$$

Substitution: $\boxed{t = \sin x}$.

Odd power of sine: $m = 2k+1$

Save one sine factor and use $\boxed{\sin^2 x = 1 - \cos^2 x}$ to express other sine factors in terms of cosine.

$$\int \sin^{2k+1} x \cos^n x \, dx = \int \left(\sin^2 x\right)^k \cos^n x \sin x \, dx = \int \left(1 - \cos^2 x\right)^k \cos^n x \sin x \, dx$$

Substitution: $\boxed{t = \cos x}$.

Even powers of sine and cosine

Use half-angle identities:

$\boxed{\sin^2 x = \tfrac{1}{2}(1 - \cos 2x)}$ or $\boxed{\cos^2 x = \tfrac{1}{2}(1 + \cos 2x)}$

Helpful identities (double-angle):

$\sin 2x = 2 \sin x \cos x$, $\cos 2x = \cos^2 x - \sin^2 x$

$\int \tan^m x \, \sec^n x \, dx$

Even power of secant: $n = 2k$

Save one factor of $\sec^2 x$, use $\boxed{\sec^2 x = 1 + \tan^2 x}$ to express other secant factors in terms of tangent.

$$\int \tan^m x \sec^n x \, dx = \int \tan^m x \left(\sec^2 x\right)^{k-1} \sec^2 x \, dx = \int \tan^m x \left(1 + \tan^2 x\right)^{k-1} \sec^2 x \, dx$$

Substitution: $\boxed{u = \tan x}$.

Odd power of tangent: $m = 2k+1$

Save a factor of $\sec x \tan x$, use $\boxed{\tan^2 x = \sec^2 x - 1}$ to express other factors in terms of secant.

$$\int \tan^{2k+1} x \sec^n x \, dx = \int \left(\tan^2 x\right)^k \sec^{n-1} x \sec x \tan x \, dx = \int \left(\sec^2 x - 1\right)^k \sec^{n-1} x \sec x \tan x \, dx$$

Substitution: $\boxed{t = \sec x}$.

Other cases: identities & integration by parts

Helpful formulas:

$$\int \tan x \, dx = \ln\left|\sec x\right| + c$$

$$\int \sec x \, dx = \ln\left|\sec x + \tan x\right| + c$$

$\int \sin mx \, \cos nx \, dx$

$\int \sin mx \, \sin nx \, dx$

$\int \cos mx \, \cos nx \, dx$

Use trigonometric identities:

$$\sin A \, \cos B = \frac{1}{2}\left[\sin(A - B) + \sin(A + B)\right]$$

$$\sin A \, \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\cos A \, \cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

Note: These identities follow from

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$