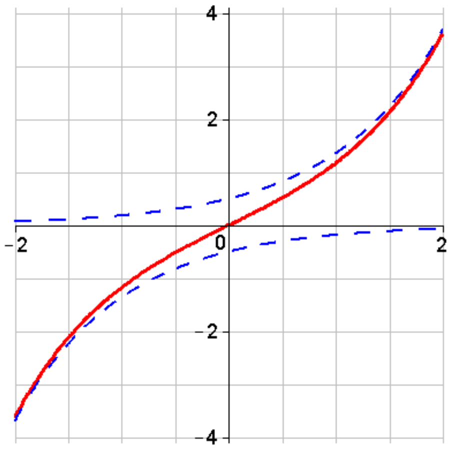
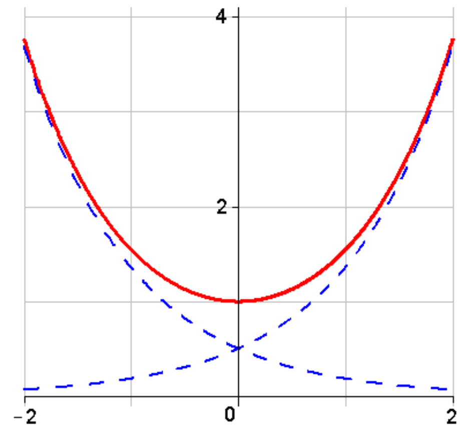
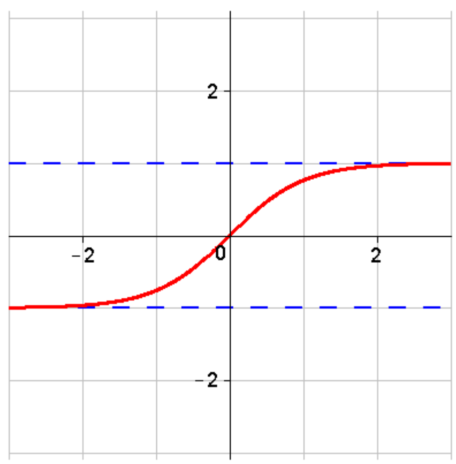
Hyperbolic sine: $\sinh x = \dfrac{e^x - e^{-x}}{2}$



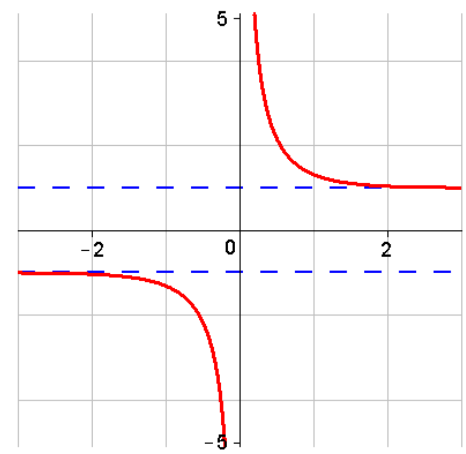
Hyperbolic cosine: $\cosh x = \dfrac{e^x + e^{-x}}{2}$



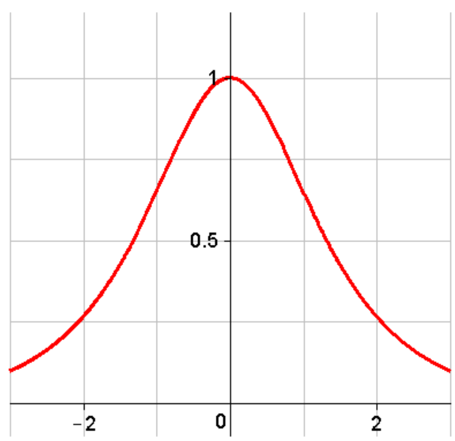
Hyperbolic tangent: $\tanh x = \dfrac{\sinh x}{\cosh x}$



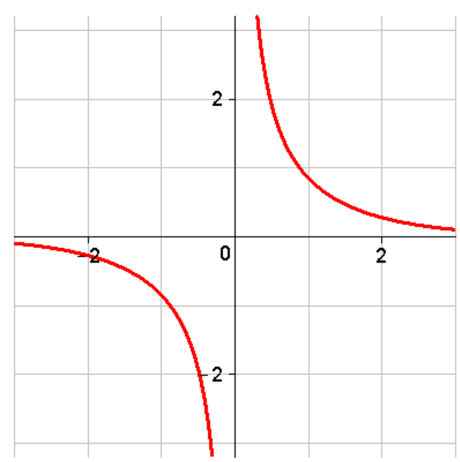
Hyperbolic cotangent: $\coth x = \dfrac{\cosh x}{\sinh x}$



Hyperbolic secant: $\operatorname{sech} x = \dfrac{1}{\cosh x}$



Hyperbolic cosecant: $\operatorname{csch} x = \dfrac{1}{\sinh x}$



Ex. Show:

a) $\sinh x = \dfrac{e^{2x}-1}{2e^{x}} = \dfrac{1-e^{-2x}}{2e^{-x}}$

b) $\cosh x = \dfrac{e^{2x}+1}{2e^{x}} = \dfrac{1+e^{-2x}}{2e^{-x}}$

c) $\tanh x = \dfrac{e^{x}-e^{-x}}{e^{x}+e^{-x}} = \dfrac{e^{2x}-1}{e^{2x}+1} = \dfrac{1-e^{-2x}}{1+e^{-2x}}$

d) $\coth x = \dfrac{e^{x}+e^{-x}}{e^{x}-e^{-x}} = \dfrac{e^{2x}+1}{e^{2x}-1} = \dfrac{1+e^{-2x}}{1-e^{-2x}}$

e) $\operatorname{sech} x = \dfrac{2}{e^{x}+e^{-x}} = \dfrac{2e^{x}}{e^{2x}+1} = \dfrac{2e^{-x}}{1+e^{-2x}}$

f) $\operatorname{csch} x = \dfrac{2}{e^{x}-e^{-x}} = \dfrac{2e^{x}}{e^{2x}-1} = \dfrac{2e^{-x}}{1-e^{-2x}}$

Useful Hyperbolic Identities

$\boxed{\sinh(-x) = -\sinh x}$ $\boxed{\cosh(-x) = \cosh x}$ $\tanh(-x) = -\tanh x$ $\coth(-x) = -\coth x$ $\operatorname{sech}(-x) = \operatorname{sech} x$ $\operatorname{csch}(-x) = -\operatorname{csch} x$	$\boxed{\cosh^2 x - \sinh^2 x = 1}$ $\operatorname{sech}^2 x = 1 - \tanh^2 x$ $\operatorname{csch}^2 x = \coth^2 x - 1$
$\boxed{\sinh(x+y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)}$ $\boxed{\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)}$ $\sinh(2x) = 2\sinh x \cdot \cosh x$ $\cosh(2x) = \cosh^2 x + \sinh^2 x = 2\sinh^2 x + 1 = 2\cosh^2 x - 1$	$\cosh x + \sinh x = e^{x}$ $\cosh x - \sinh x = e^{-x}$

Derivatives of Hyperbolic Functions

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

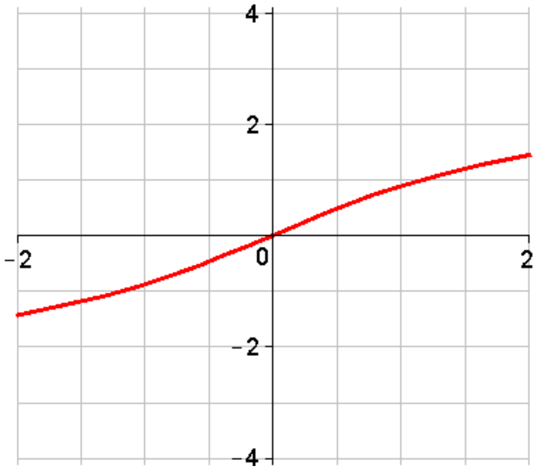
$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \cdot \tanh x$$

$$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \cdot \coth x$$

Inverse Hyperbolic Functions

Inverse sinh *x*

$$f(x) = \sinh^{-1} x$$



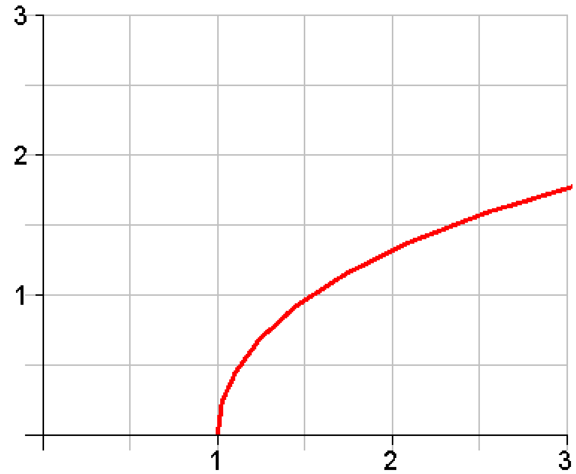
$D_f = (-\infty, \infty)$ and $R_f = (-\infty, \infty)$

$\sinh^{-1}(\sinh x) = x$ for all x

$\sinh(\sinh^{-1} x) = x$ for all x

Inverse cosh *x*

$$f(x) = \cosh^{-1} x$$



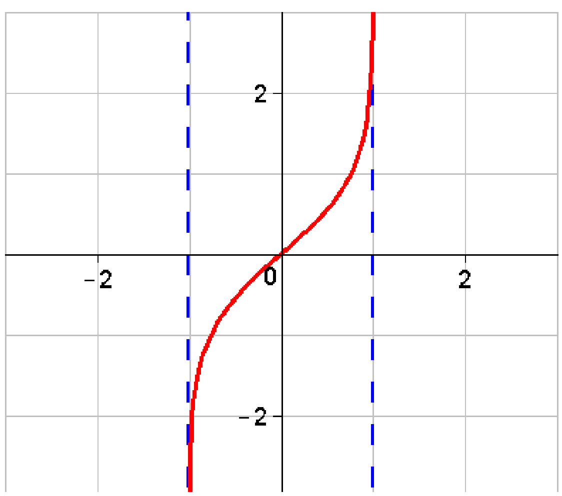
$D_f = [1, \infty)$ and $R_f = [0, \infty)$

$\cosh^{-1}(\cosh x) = x$ if $x \in [0, \infty)$

$\cosh(\cosh^{-1} x) = x$ if $x \in [1, \infty)$

Inverse tanh *x*

$$\tanh^{-1} x$$



$D_f = (-1, 1)$

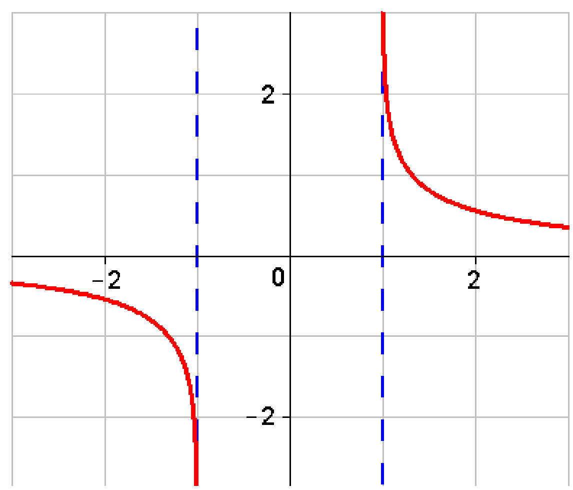
$R_f = (-\infty, \infty)$

$\tanh^{-1}(\tanh x) = x$ for all x

$\tanh(\tanh^{-1} x) = x$ for $x \in (-1, 1)$

Inverse coth *x*

$$f(x) = \coth^{-1} x$$



$D_f = (-\infty, -1) \cup (1, \infty)$

$R_f = (-\infty, 0) \cup (0, \infty)$

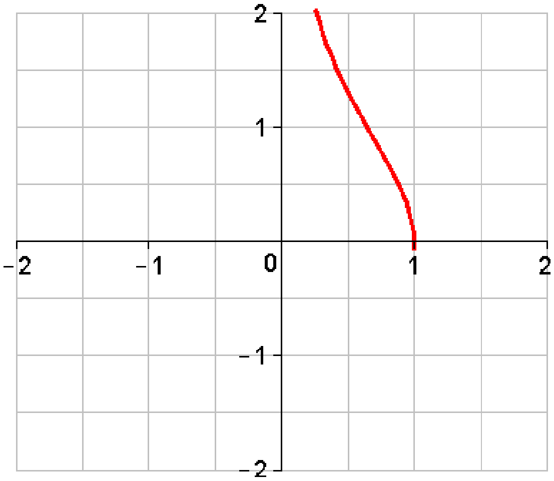
$\coth^{-1}(\coth x) = x$ for $x \in (-\infty, 0) \cup (0, \infty)$

$\coth(\coth^{-1} x) = x$ for $x \in (-\infty, -1) \cup (1, \infty)$

Inverse sech *x*

Inverse csch *x*

$f(x) = \text{sech}^{-1} x$



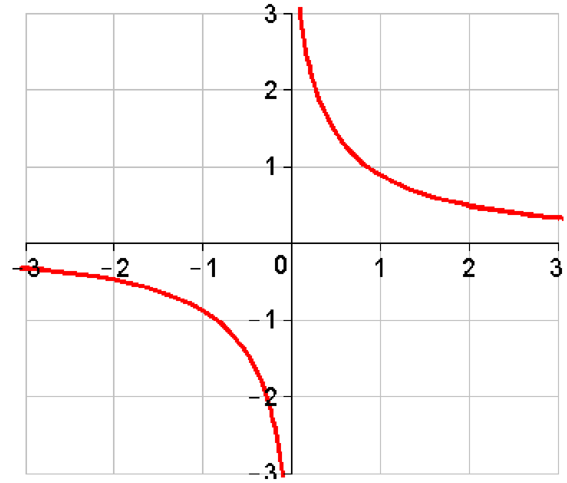
$D_f = (0,1]$

$R_f = [0,\infty)$

$\sec h^{-1}(\text{sech}\, x) = x \quad \text{for} \quad x \in [0,\infty)$

$\text{sech}(\text{sech}^{-1} x) = x \quad \text{for} \quad x \in (0,1]$

$f(x) = \text{csch}^{-1} x$



$D_f = (-\infty,0) \cup (0,\infty)$

$R_f = (-\infty,0) \cup (0,\infty)$

$\text{csch}^{-1}(\text{csch}\, x) = x \quad \text{for} \quad x \in (-\infty,0) \cup (0,\infty)$

$\text{csch}(\text{csch}^{-1} x) = x \quad \text{for} \quad x \in (-\infty,0) \cup (0,\infty)$

Domain	Inverse hyperbolic functions as LOGARITHMS	DERIVATIVES of inverse hyperbolic functions
$x \in \mathbb{R}$	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$	$\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{x^2+1}}$
$x \geq 1$	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$	$\frac{d}{dx}\left(\cosh^{-1} x\right) = \frac{1}{\sqrt{x^2-1}}$
$\lvert x \rvert < 1$	$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$	$\frac{d}{dx}\left(\tanh^{-1} x\right) = \frac{1}{1-x^2}$
$\lvert x \rvert > 1$	$\coth^{-1} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$	$\frac{d}{dx}\left(\coth^{-1} x\right) = \frac{1}{1-x^2}$
$0 < x \leq 1$	$\text{sech}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1-x^2}}{x}\right)$	$\frac{d}{dx}\left(\text{sech}^{-1} x\right) = -\frac{1}{x\sqrt{1-x^2}}$
$x \neq 0$	$\text{csch}^{-1} x = \ln\left(\frac{1}{\lvert x \rvert} + \frac{\sqrt{1+x^2}}{\lvert x \rvert}\right)$	$\frac{d}{dx}\left(\text{csch}^{-1} x\right) = -\frac{1}{\lvert x \rvert\sqrt{1+x^2}}$

INTEGRALS whose solutions are inverse hyperbolic functions

$$\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{x^2+1}} \quad \Rightarrow \quad \boxed{\int \frac{dx}{\sqrt{x^2+1}} = \sinh^{-1} x + C} \quad , \quad x \in \mathbb{R}$$

$$\frac{d}{dx}\left(\cosh^{-1} x\right) = \frac{1}{\sqrt{x^2-1}} \quad \Rightarrow \quad \boxed{\int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1} x + C} \quad , \quad x \geq 1$$

$$\frac{d}{dx}\left(\tanh^{-1} x\right) = \frac{1}{1-x^2} \quad \Rightarrow \quad \boxed{\int \frac{dx}{1-x^2} = \tanh^{-1} x + C} \quad , \quad |x| < 1$$

$$\frac{d}{dx}\left(\coth^{-1} x\right) = \frac{1}{1-x^2} \quad \Rightarrow \quad \boxed{\int \frac{dx}{1-x^2} = \coth^{-1} x + C} \quad , \quad |x| > 1$$

$$\frac{d}{dx}\left(\operatorname{sech}^{-1} x\right) = -\frac{1}{x\sqrt{1-x^2}} \quad \Rightarrow \quad \boxed{\int \frac{dx}{x\sqrt{1-x^2}} = -\operatorname{sech}^{-1} x + C} , \quad 0 < x \leq 1$$

$$\frac{d}{dx}\left(\operatorname{csch}^{-1} x\right) = -\frac{1}{|x|\sqrt{1+x^2}} \quad \Rightarrow \quad \boxed{\int \frac{dx}{|x|\sqrt{1+x^2}} = -\operatorname{csch}^{-1} x + C} , \quad x \neq 0$$

More general forms:

$$\int \frac{du}{\sqrt{u^2+a^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C \quad , \quad u \in \mathbb{R}$$

$$\int \frac{dx}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C \quad , \quad u^2 \geq a^2$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C \quad , \quad u^2 < a^2$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C \quad , \quad u^2 > a^2$$

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C \quad , \quad 0 < u^2 \leq a^2$$

$$\int \frac{du}{|u|\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left(\frac{u}{a}\right) + C \quad , \quad u \neq 0$$