§11.5 Alternating Series

ALTERNATING SERIES	- series whose *terms alternate in sign* Given $b_n > 0$, we can build an alternating series multiplying b_n by $(-1)^n$ or $(-1)^{n-1}$, so: $\sum_{n=1}^{\infty}(-1)^n b_n = -b_1 + b_2 - b_3 + \dots$ - this series starts with a NEGATIVE term $\sum_{n=1}^{\infty}(-1)^{n-1} b_n = b_1 - b_2 + b_3 - \dots$ - this series starts with a POSITIVE term
ALTERNATING SERIES TEST	An alternating series converges if the terms decrease toward 0 in their *absolute values*. If the alternating series: $\sum_{n=1}^{\infty}(-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \dots \;,\quad b_n > 0$ satisfies: 1. $b_{n+1} \le b_n$, for all n (or eventually) 2. $\lim_{n\to\infty} b_n = 0$ } by their absolute value, the terms are decreasing and approaching zero then the series is CONVERGENT.
ESTIMATING SUMS	– To estimate the sum s of a *convergent* series, we use the partial sum s_n: $\boxed{s \approx s_n}$ – The accuracy of the approximation is estimated from the remainder: $\boxed{R_n = s - s_n}$ **Alternating Series Estimation Theorem**: For a convergent alternating series whose sum is $s = \sum_{n=1}^{\infty}(-1)^{n-1} b_n$, the error of estimating s using the n-th partial sum is smaller than the absolute value of the 1st neglected term: $\boxed{\lvert R_n \rvert = \lvert s - s_n \rvert \le b_{n+1}}$ s - sum of the series s_n - n-th partial sum b_{n+1} - absolute value of the first neglected term