## §11.2 Series

INFINITE SERIES  
(series)-THE SUM of the terms of an infinite sequence 
$$\{a_n\}_{n=1}^{\infty}$$
  
Notation:  $\sum_{n=1}^{\infty} a_n$  or  $\sum a_n$ PARTIAL SUM $\overline{s_n} = \sum_{k=1}^n a_k = a_1 + a_2 + \dots + a_n$   
 $s_1 = \sum_{k=1}^{1} a_k = a_1$   
 $s_2 = \sum_{k=1}^{1} a_k = a_1 + a_2$   
 $s_2 = \sum_{k=1}^{1} a_k = a_1 + a_2 + a_3$   
 $\vdots$ Given:  
 $1.$  series  $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$   
 $2.$  sequence of partial sums  $s_n = \sum_{k=1}^n a_k = a_1 + a_2 + a_3$   
 $\vdots$ Given:  
 $1.$  series  $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$   
 $2.$  sequence of partial sums  $s_n = \sum_{k=1}^n a_k = a_1 + a_2 + \dots + a_n$   
If the sequence  $\{s_n\}$  is convergent and  $\lim_{n \to \infty} s_n = s$ , then  
 $\sum_{n=1}^{\infty} a_n = \lim_{n \to \infty} \sum_{k=1}^n a_k = \lim_{n \to \infty} s_n = s$ Sum of a  
series-THE LIMIT OF THE SEQUENCE OF PARTIAL SUMS: $\sum_{n=1}^{\infty} a_n = \lim_{n \to \infty} \sum_{k=1}^n a_k = \lim_{n \to \infty} s_n = s$ CONVERGENT  
series- a series whose sequence of partial sums  $\{s_n\}$  DIVERGES- a series whose sequence of partial sums  $\{s_n\}$  DIVERGESTHEOREMIf the series  $\sum_{n=1}^{\infty} a_n$  is convergent, then  $\lim_{n \to \infty} a_n = 0$ .If the series  $\sum_{n=1}^{\infty} a_n$  is DIVERGENT  
DIVERGENTTHEOREMIf the series  $\sum_{n=1}^{\infty} a_n$  is convergent, then  $\lim_{n \to \infty} a_n = 0$ .

Note: with a series  $\sum a_n$  , we associate two sequences:

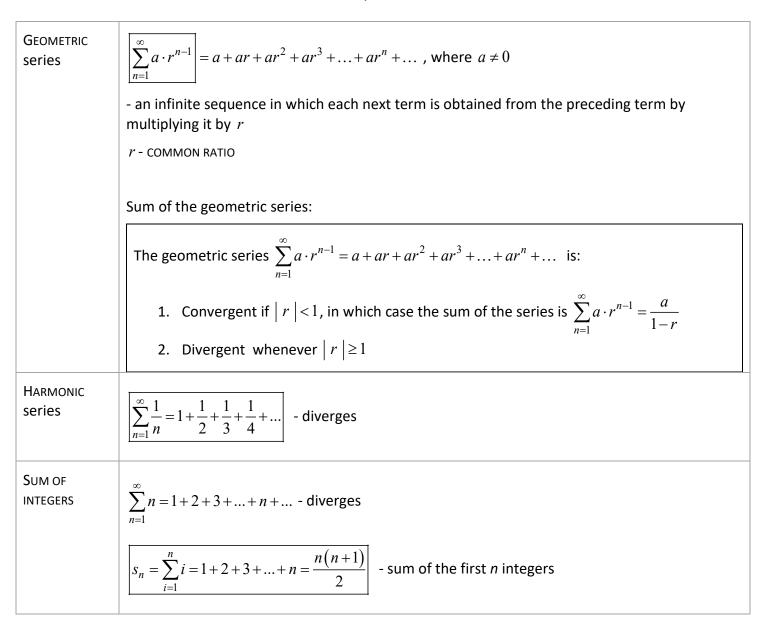
1.  $\{s_n\}$  - sequence of partial sums

2.  $\{a_n\}$  - sequence of the terms of the series

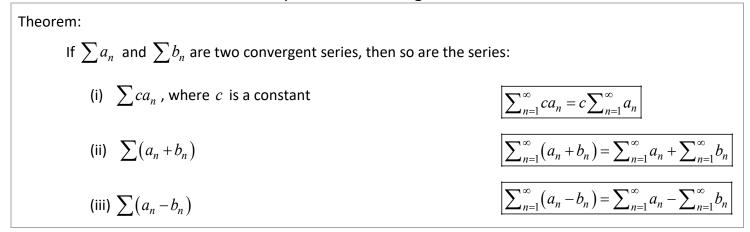
If a series is convergent, then the sum is the limit of the sequence of partial sums and also  $\lim_{n\to\infty} a_n = 0$ .

The converse does not hold: if  $\lim_{n\to\infty} a_n = 0$  we cannot conclude anything about the convergence of the series However, if we find  $\lim_{n \to \infty} a_n \neq 0$  , then the series is divergent.

## Some Important series



## **Properties of Convergent Series**



*Note*: A finite number of terms do not affect the convergence or divergence of a series.