§10.4 Areas and Lengths in Polar Coordinates

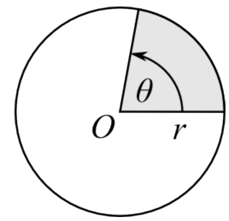
<table>
<tr><td>AREA OF A SECTOR OF A CIRCLE</td><td>

Recall the formula for the area for a sector of a circle:

$$A = \frac{1}{2} r^2 \theta$$

r - the radius

θ - the radian measure of the central angle



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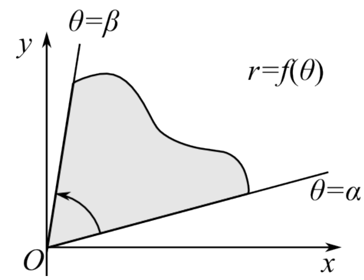
<tr><td>AREA BOUNDED BY THE POLAR CURVE
$r = f(\theta)$</td><td>

The area of a region bounded by a polar curve $r = f(\theta)$ and by the rays $\theta = \alpha$ and $\theta = \beta$ is:

$$\boxed{A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta},$$



Note: remember that $r(\theta)$ is a function of θ.

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<tr><td>ARC LENGTH</td><td>

The length of a polar curve $r = f(\theta)$, $a \le \theta \le b$ is obtained from

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \, d\theta$$

Since we have

$$x = r(\theta)\cos\theta \quad \Rightarrow \quad \frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta$$

$$y = r(\theta)\sin\theta \quad \Rightarrow \quad \frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$$

the expression for the length of a polar curve can be written as:

$$\boxed{L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta}$$

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