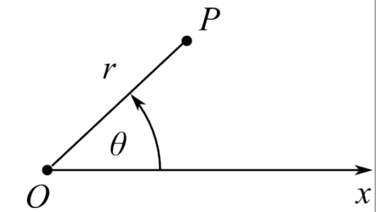
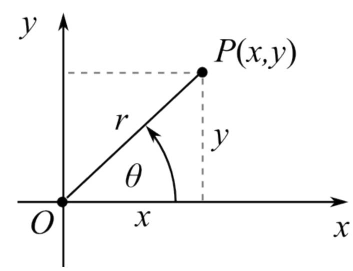
§10.3 Polar Coordinates

Polar Coordinate System	…is a coordinate system with its origin at a pole *O* and a polar axis from which we measure the polar angle. In this system a point *P* is represented by a pair of numbers (r,θ) where:  r - **radius**, distance from the pole *O* to the point *P* θ - **polar angle**, angle between the polar axis and the line *OP* Note: **positive angle** is measured in *counterclockwise* direction from the polar axis
Coordinates of the Pole	For the pole $r=0$, so the pole is represented by the point $(0,\theta)$, for any value of θ.
Negative Radius r	The points (r,θ) and $(-r,\theta)$ lie on the same line through *O* and have the same distance $\lvert r\rvert$ from *O*, but they are located on the opposite sides of *O*. Hence ▫ $r>0$ means the point (r,θ) lies in the same quadrant as θ ▫ $r<0$ means the point (r,θ) lies in the quadrant on opposite side of the pole *O*
Notation	$(r,\theta)=(r,\theta+2n\pi)=(-r,\theta+(2n+1)\pi)$
Cartesian and Polar Coordinates	To obtain the Cartesian coordinates from polar coordinates: $\boxed{\begin{aligned} x&=r\cos\theta \\ y&=r\sin\theta \end{aligned}}$ To obtain polar coordinates from Cartesian coordinates: $\boxed{\begin{aligned} r&=\sqrt{x^2+y^2} \\ \tan\theta&=\frac{y}{x} \end{aligned}}$ 
Polar Curve	Equation of a polar curve (polar equation): $\boxed{r=f(\theta)}$ or $\boxed{F(r,\theta)=0}$
Symmetry	If a polar equation does not change when: ▫ θ is replaced by $-\theta$, the curve is symmetric about the <u>polar axis</u> ▫ r is replaced by $-r$ or if θ is replaced by $\theta+\pi$, the curve is symmetric about the <u>pole</u> ▫ θ is replaced by $\pi-\theta$ <u>vertical line</u> $\theta=\pi/2$
Tangents to Polar Curves	To find a tangent line to a polar curve $r=f(\theta)$, write its parametric equations a $x=r\cos\theta=f(\theta)\cos\theta$ and $y=r\sin\theta=f(\theta)\sin\theta$ Then $\dfrac{dy}{dx}=\dfrac{dy}{dx}=\dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\boxed{\dfrac{\frac{dr}{d\theta}\cdot\sin\theta+r\cos\theta}{\frac{dr}{d\theta}\cdot\cos\theta-r\sin\theta}}$ Horizontal tangents: $dy/d\theta=0$, provided that $dx/d\theta\neq 0$ Vertical tangents: $dx/d\theta=0$, provided that $dy/d\theta\neq 0$.